

## Homework of Week 4

**Deadline: 9:00am, October 21 (Thursday), 2021**

1. Consider the following generalization of the set balancing problem. There are  $m$  objects and  $n$  features, with each object having some features. The objective is to partition the objects into two groups  $G_1$  and  $G_2$  such that every feature is 3 : 1 divided, i.e., for every feature, the number of objects in  $G_1$  having the feature is triple the number of objects in  $G_2$  having the feature. Please design an efficient randomized algorithm for this task, and show that the error is small with high probability when  $m$  and  $n$  are sufficiently large.

Hint: Obviously, the domain of the vector  $b$  should not be  $\{-1, 1\}^m$ . The point is to find a suitable domain.

2. Suppose  $X$  is a Bernoulli random variable with  $\Pr(X = 1) = \frac{1}{2}$ . Calculate  $I(\frac{5}{6}) \triangleq \sup_{\lambda > 0} \frac{5\lambda}{6} - \ln \mathbb{E}[e^{\lambda X}]$ . Compare the result with 0.2426.
3. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string  $s_1 s_2 \dots s_i \dots s_{20}$ , where  $s_i$  is 1 if the  $i^{\text{th}}$  trial gets Head, and otherwise is 0.