Homework of Week 12

Deadline: 9:00am, December 16 (Thursday), 2021

1. Prove that, for every integer n, there exists a way to 2-color the edges of K_x so that there is no monochromatic clique of size k when $x = n - \binom{n}{k}2^{1-\binom{k}{2}}$. Note that K_x stands for the x-vertex complete graph. (Hint, start by 2-coloring the edges of K_n and fix things up.)

2. (**Optional. Bonus score 5 points**) Page 12 of Lecture 12 presents a randomized algorithm to find a graph with big chromatic number and big girth. Can you derive an efficient deterministic algorithm from it?

3. Consider a graph in $G_{n,p}$ with $p = c\frac{\ln n}{n}$. Use the second moment method to prove that if $c < 1$ then, for any constant $\epsilon > 0$ and for sufficiently large n, the graph has isolated vertices with probability at least $1 - \epsilon$.

4. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the ith trial gets Head, and otherwise is 0.